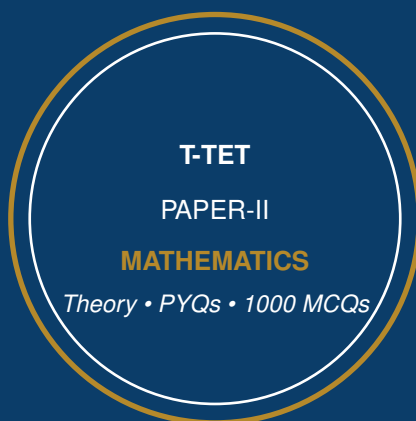


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THE COMPLETE T-TET PAPER-II

MATHEMATICS

COMPLETE EDITION — VOLUMES 1–3 IN ONE BOOK



Vol 1 Theory & Practice • Vol 2 All PYQs Solved • Vol 3 1000 Hard MCQs
Built on 8 years of PYQ analysis • NCERT-aligned • 2025-pattern rigour

SUDIP LASKAR

CSIR-NET JRF (AIR 93, 162) • GATE Physics AIR 52 • STPGT Physics Rank 2

The Complete T-TET Paper-II

MATHEMATICS — Complete Edition

Volume 1 (Theory & Practice) | Volume 2 (All PYQs Solved) | Volume 3 (1000 Hard MCQs)

First Edition

Sudip Laskar

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Disclaimer. This complete edition combines all three volumes of the series in a single, continuously-paginated book. Previous-year questions are reproduced from publicly conducted T-TET examinations for educational and analytical purposes only, and have been re-typeset, solved and explained by the author. Where an official answer key is mathematically indefensible, the scientifically correct answer is given with reasoning and the discrepancy flagged. The 1000 practice questions of Volume 3 are entirely original.

Preface

The Tripura Teacher Eligibility Test (T-TET) Paper-II is the gateway to an upper-primary teaching career, and its 30-question Mathematics component rewards candidates who combine sound concepts with fast, accurate problem-solving. This *Complete Edition* brings the whole series together in one book so that a candidate can move seamlessly from learning a topic, to seeing every way the board has ever tested it, to drilling it to mastery.

The book is organised in three Parts. **Part I (Volume 1)** teaches every syllabus topic with complete theory, worked examples, the chapter's previous-year questions and graded practice. **Part II (Volume 2)** reproduces and fully solves *every* Mathematics question from the 2015–2025 papers, chapter by chapter. **Part III (Volume 3)** is a bank of 1000 fresh, hard practice questions — none repeated from the papers or from each other — with worked answer keys. Everything is built on a question-by-question analysis of the actual papers, and everything is pitched at (or above) the sharply harder 2025 standard.

— Sudip Laskar, for Quarks Institute

How to Use This Book

Use the three Parts as a cycle. **Learn** a chapter from Part I; **test** yourself against its real previous-year questions in Part II; then **drill** the matching chapter of Part III until the methods are automatic. The unified *Contents* that follows lists every chapter of all three Parts with its page number, so you can jump straight to any topic. Each Part's chapters follow the syllabus in the same order, so "Chapter 8" means *Algebraic Expressions and Identities* in every Part.

Finding things fast

The **Contents** (next page) is your master index: Part I chapters are the theory, Part II the same chapters solved from the papers, Part III the same chapters as fresh practice. Page numbers run continuously through the whole book.

Contents

Complete index of all three volumes — chapter name and page number.

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PART I

Volume I — Theory, Solved Examples & Practice

PYQ Analysis Report — Mathematics

This report is the analytical foundation of the entire series. It is built on the **eight official T-TET Paper-II papers** available to us — **2015, 2016, 2017, 2018 (Session-1), 2018 (Session-2), 2019, 2022 and 2025** — covering roughly **237 Mathematics questions**. Every chapter's depth, number of examples and quantity of practice was decided by what you are about to see.

1. How the Mathematics section is set

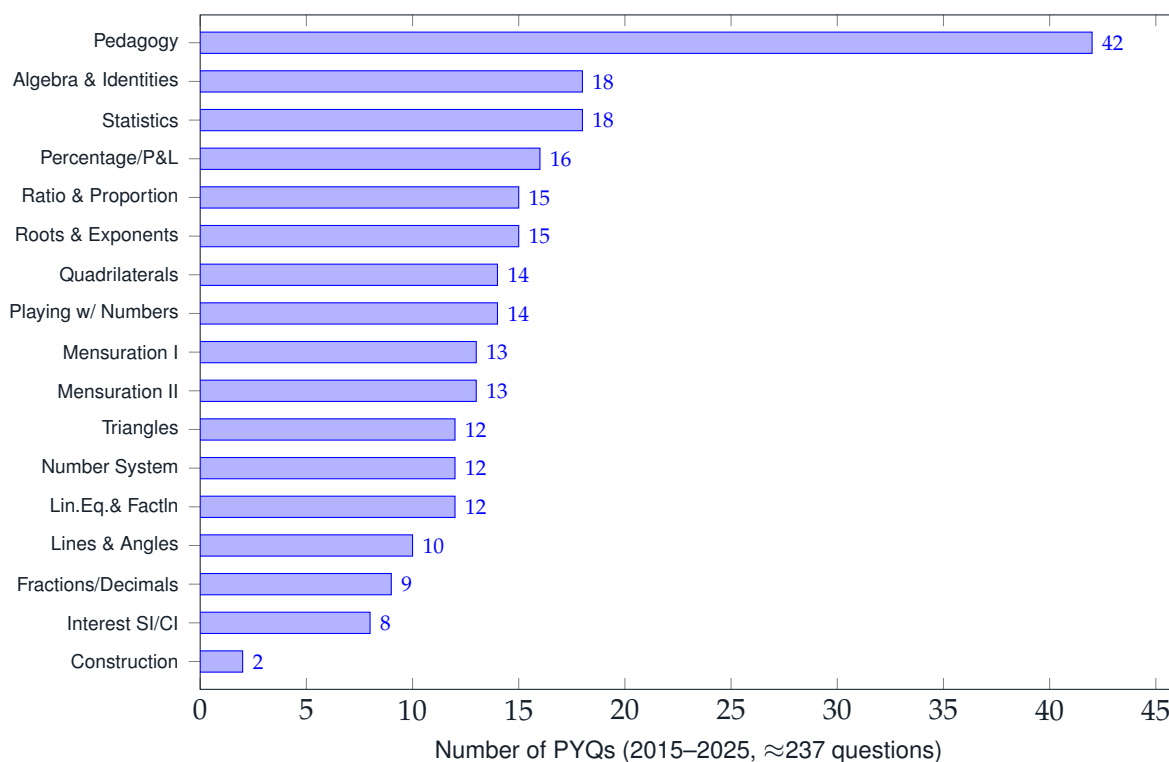
Exam Structure You Must Know

In T-TET Paper-II, Mathematics and Science together form a 60-question part. **Mathematics is 30 questions, numbered Q91–Q120** (the 2025 paper ran pure mathematics Q91–Q114 and mathematics-pedagogy Q115–Q117). Each question is a four-option MCQ; the paper is **bilingual** (English left column, Bengali right). A recurring stylistic feature: option **(D) is very often “None of the above”**, and in several papers the “correct” printed key is actually (D) because options (A)–(C) are near-misses. Train for this.

2. Chapter-wise weightage (the master table)

The table below shows the approximate concept-wise distribution across the 237 questions (a few questions touch two areas, so totals are indicative rather than exact). **The last column is your revision priority.**

#	Chapter / Topic	PYQs	%	Priority
1	The Number System	12	5.0	High
2	Playing with Numbers (divisibility, digits)	14	5.9	High
3	Fractions and Decimals	9	3.8	Medium
4	Squares, Cubes, Roots & Exponents	15	6.3	Very High
5	Ratio, Proportion & Variation	15	6.3	Very High
6	Percentage, Profit & Loss	16	6.8	Very High
7	Simple & Compound Interest	8	3.4	Medium
8	Algebraic Expressions & Identities	18	7.6	Very High
9	Linear Equations & Factorisation	12	5.0	High
10	Basic Geometry — Lines & Angles	10	4.2	High
11	Triangles & Congruency	12	5.0	High
12	Quadrilaterals, Polygons & Symmetry	14	5.9	Very High
13	Practical Geometry (Construction)	2	0.8	Low
14	Mensuration I — Perimeter & Area	13	5.5	High
15	Mensuration II — Surface Area & Volume	13	5.5	High
16	Data Handling & Statistics	18	7.6	Very High
17	Pedagogy of Mathematics	42	17.7	Critical
Total (indicative)		243	100	



Read the chart like a strategist

Just **five** blocks — *Pedagogy, Algebraic Identities, Statistics, Percentage/Profit–Loss and Ratio–Proportion* — account for well over **45%** of all Mathematics marks. If your time is short, these are non-negotiable. **Pedagogy alone is 5–6 sure questions in every single paper**; it is the highest return-on-effort topic in the book and is treated at length in Chapter 17.

3. Frequently repeated concepts (the board's favourites)

The same ideas return year after year, only with the numbers changed. Master these patterns and you have already answered a large share of any future paper:

- **The $x + \frac{1}{x}$ ladder** — given $x + \frac{1}{x}$, find $x^2 + \frac{1}{x^2}$, $x^3 + \frac{1}{x^3}$, even $x^5 + \frac{1}{x^5}$. (2017 Q112, 2018-S2 Q104/Q109, 2019 Q95, 2022 Q118.)
- **Empirical Mode = 3 Median – 2 Mean** — asked directly and indirectly almost every year (2015 Q114, 2017 Q96, 2018-S2 Q103, 2019 Q115, 2025 Q91).
- **Angle bisectors of a triangle / parallelogram** — $\angle BOC = 90^\circ + \frac{1}{2}\angle A$ and its cousins (2015 Q105, 2017 Q93, 2025 Q95).
- **Cylinder CSA = 264, Volume = 924** — the identical numbers appear in 2015 (find height) and 2022 (find diameter : height). A gift if you have seen it once.
- **Successive vs. single discount, and the dishonest-dealer gain** (2016 Q97, 2017 Q108/Q109).
- **Surd simplification** $\sqrt{3 \pm 2\sqrt{2}}$, $x = 5 + 2\sqrt{6}$ (2018-S2 Q112, 2019 Q101).
- **Euler's polyhedron formula** $F + V - E = 2$ / counting vertices, edges, faces (2017 Q94, 2022 Q97, 2025 Q92).
- **Divisibility by 72 (i.e. by 8 and 9)** of a number with unknown digits (2025 Q98; CTET-bank twins).

4. Conceptual traps and common mistakes

The five traps that cost the most marks

(1) **The “None of the above” trap:** several correct answers are (D) because the paper’s own arithmetic in (A)–(C) is wrong; always compute, never pattern-match. (2) **Percentage base errors:** “ $x\%$ more” then “ $y\%$ less” is *not* zero change — successive-percentage must be chained. (3) **Mode–median–mean:** students misremember the empirical formula as $2 \text{ Median} - 3 \text{ Mean}$. It is $3 \text{ Median} - 2 \text{ Mean}$. (4) **Volume/area scaling:** scaling a length by k scales area by k^2 and volume by k^3 (the deflated-balloon question). (5) **Rupee–paise and unit slips** in SI/CI and speed questions.

5. Difficulty trend — and why this series is deliberately hard

Papers from 2015–2019 sat at a comfortable upper-primary level. The **2022 and especially the 2025 paper raised the bar sharply** — multi-step algebra, nested surds, polyhedra, ogives and genuinely Olympiad-flavoured reasoning. In line with your instruction and the real trend, **this series contains no trivial questions**. Every solved example and every practice item is pitched at the 2025 standard or above (strong upper-primary / Class 9–10 / Olympiad-flavoured), so that the actual exam feels comfortable.

6. Most probable future questions

Based on the frequency and recency analysis, the highest-probability 2026 topics are: (i) a mode/median/mean empirical-relation item; (ii) an $x \pm \frac{1}{x}$ identity; (iii) a cylinder/cone/sphere mensuration item; (iv) a successive-discount or dishonest-dealer profit item; (v) a triangle/quadrilateral angle-chase; and (vi) 5–6 pedagogy questions spanning great mathematicians, teaching methods, CCE/assessment and terms such as *dyscalculia*, *NCTM*, *TLM* and *MKO*. Each is drilled in the relevant chapter.

One-line takeaway

Learn **Pedagogy** cold, own the **identity/statistics/percentage** engines, and be fluent in **mensuration and angle-chasing**. That is where the paper lives.

CHAPTER 1

The Number System

LEARNING OBJECTIVES

After working through this chapter you will be able to:

- classify any number as natural, whole, integer, rational or irrational and place it correctly in the hierarchy $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$;
- state *and use* closure, commutativity, associativity, distributivity, identity and inverse, and quote a counterexample for every failure;
- exploit the *density* of rationals — insert one or n rationals between two numbers by the average, common-denominator and step methods;
- decide from the denominator alone whether a fraction terminates ($q = 2^m 5^n$) and convert both ways, including *mixed* recurring decimals;
- reason about irrationals (why $\sqrt{2}$ is irrational; what stays rational under $+$, $-$, $\times$, $\div$);
- count integers in an inequality range, handle absolute value, and write and combine numbers in standard (scientific) form $k \times 10^n$ — all at 2025 T-TET / junior-Olympiad difficulty.

PYQ TREND ANALYSIS

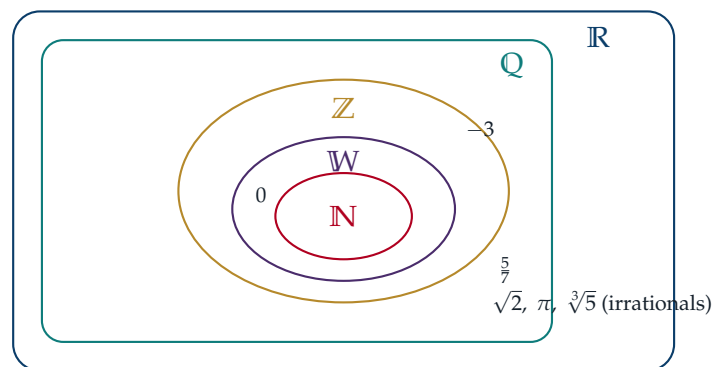
PYQs from this chapter: ≈ 12 across 2015–2025. **Frequently asked:** inserting a rational number between two given numbers; the product or count of integers in a range; properties (which set is “closed” under an operation); recurring decimal $\leftrightarrow$ fraction; standard/scientific form and the combination $k + n$. **Probability of a question in 2026: very high (≥ 1 , often 2)** — expect one “insert a rational” or “count the integers” item and one properties- or standard-form item. The examiner’s favourite trap is the boundary of a strict versus non-strict inequality and the sign of a product of negatives.

1.1 The family of numbers

Every number in the syllabus belongs to one nested family. Knowing the hierarchy instantly kills a whole class of MCQs, because a claim that holds in a larger set need not hold in a smaller one.

The number hierarchy $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

Natural $\mathbb{N} = \{1, 2, 3, \dots\}$ — the counting numbers. **Whole** $\mathbb{W} = \{0, 1, 2, \dots\}$ — adds 0. **Integers** $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ — adds the negatives. **Rational** $\mathbb{Q} = \left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$ — every terminating or recurring decimal. **Irrational** $\mathbb{Q}'$ — non-terminating, non-recurring (e.g. $\sqrt{2}$, $\sqrt[3]{7}$, π , $0.1010010001\dots$). $\mathbb{Q} \cup \mathbb{Q}' = \mathbb{R}$, the **real** numbers, which exactly fill the number line.



Read the diagram like a checklist

A number is natural $\Rightarrow$ it is automatically whole, integer, rational and real. Going *outwards* each ring only *adds* numbers. So $-3 \in \mathbb{Z}$ but $-3 \notin \mathbb{W}$; $0 \in \mathbb{W}$ but $0 \notin \mathbb{N}$; $\frac{5}{7} \in \mathbb{Q}$ but $\frac{5}{7} \notin \mathbb{Z}$; $\sqrt{2} \in \mathbb{R}$ but $\sqrt{2} \notin \mathbb{Q}$. “Is $\sqrt{4}$ irrational?” No — $\sqrt{4} = 2$ is a perfect square, hence natural.

Perfect powers hide inside the surd

$\sqrt{2}, \sqrt{3}, \sqrt{5}$ are irrational, but $\sqrt{16}, \sqrt{2.25}, \sqrt[3]{-27}$ are *rational* ($= 4, 1.5, -3$). Never label a number irrational merely because a root sign is printed; simplify first. Only the root of a *non-perfect* power is irrational.

1.2 Properties of operations (the “closure” questions)

For all a, b, c in the stated set the following may or may not hold. The right-hand column — *where the property breaks* — is what the paper actually tests.

Property	Statement	Where it fails (counterexample / trap)
Closure	result stays in the set	$\mathbb{N}$ <i>not</i> closed under $-$ ($2 - 5 = -3 \notin \mathbb{N}$) or $\div$ ($2 \div 3 \notin \mathbb{N}$)
Commutative	$a + b = b + a, ab = ba$	<i>fails</i> for $-$ ($5 - 2 \neq 2 - 5$) and $\div$ ($6 \div 2 \neq 2 \div 6$)
Associative	$(a + b) + c = a + (b + c)$	<i>fails</i> for $-$ and $\div$: $(8 - 3) - 2 = 3 \neq 8 - (3 - 2) = 7$
Distributive	$a(b + c) = ab + ac$	of $\times$ over $+$ only; <i>not</i> $a + (bc) = (a + b)(a + c)$
Additive identity	$a + 0 = a$	$0 \in \mathbb{W}$ but $0 \notin \mathbb{N}$, so $\mathbb{N}$ has no additive identity
Multiplic. identity	$a \cdot 1 = a$	1 lies in every set from $\mathbb{N}$ up
Additive inverse	$a + (-a) = 0$	exists in $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$; <i>not</i> in $\mathbb{N}, \mathbb{W}$
Multiplic. inverse	$a \cdot \frac{1}{a} = 1$ ($a \neq 0$)	exists in $\mathbb{Q} \setminus \{0\}, \mathbb{R} \setminus \{0\}$; in $\mathbb{Z}$ only ± 1 ; 0 <i>never</i>

Instant elimination

“Which set is closed under *subtraction*?” $\Rightarrow$ not $\mathbb{N}$, not $\mathbb{W}$; the answer is $\mathbb{Z}$ (or $\mathbb{Q}, \mathbb{R}$). “Smallest whole number” = 0; “smallest natural” = 1; there is **no** largest in either. 0 has *no* multiplicative inverse; only $+1$ and -1 are their own reciprocals in $\mathbb{Z}$.

Closure table at a glance

$\mathbb{N}$: closed under $+$, $\times$ only. $\mathbb{W}$: closed under $+$, $\times$ only. $\mathbb{Z}$: closed under $+$, $-$, $\times$ (*not* $\div$). $\mathbb{Q}$ and $\mathbb{R}$: closed under $+$, $-$, $\times$ and $\div$ (dividing by 0 excluded). Irrationals $\mathbb{Q}'$: *not* closed under any of $+$, $-$, $\times$, $\div$ — e.g. $\sqrt{2} \cdot \sqrt{2} = 2$, $\sqrt{2} - \sqrt{2} = 0$ are rational.

“Closed” means every result, not one

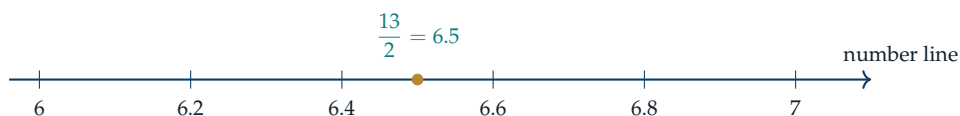
A set is closed under an operation only if the result stays in the set for *all* choices. One escape is enough to break closure: because $3 - 5 = -2 \notin \mathbb{W}$, $\mathbb{W}$ is not closed under subtraction — even though $5 - 3 = 2$ is fine. Do not be fooled by a single friendly example.

1.3 The number line and the density of rationals

Density — the single most tested idea here

Between *any* two distinct rationals there lie **infinitely many** rationals. The quickest one to name is the **average**: between a and b lies $\frac{a+b}{2}$, and you can repeat this forever. To insert n rationals between a and b use either method:

- **Common denominator** — scale a, b so their numerators differ by more than n , then read off the in-between numerators.
- **Step method** — the equally spaced values $a + k \cdot \frac{b-a}{n+1}$ for $k = 1, 2, \dots, n$.



A rational between 6 and 7 is simply their average $\frac{6+7}{2} = \frac{13}{2} = 6.5$ — the exact 2019 board question (see PYQ below). Note $\frac{12}{2} = 6$ and $\frac{14}{2} = 7$ are the *endpoints*, not strictly between; that is the trap.

Between two rationals lies an irrational too

Density is not a monopoly of the rationals: between any two rationals there is also an irrational, and between any two irrationals a rational. So both $\mathbb{Q}$ and $\mathbb{Q}'$ are “dense” in $\mathbb{R}$. A quick irrational between a and b : take $a + \frac{b-a}{\sqrt{2}}$.

Comparing two fractions fast

To compare $\frac{p}{q}$ and $\frac{r}{s}$ (positive q, s), cross-multiply: $\frac{p}{q} \leq \frac{r}{s} \iff ps \leq rq$. Example: $\frac{7}{11}$ vs $\frac{2}{3}$: $7 \cdot 3 = 21$ against $2 \cdot 11 = 22$; since $21 < 22$, $\frac{7}{11} < \frac{2}{3}$. No decimals, no LCM.

1.4 Decimals: terminating vs. recurring

When does a fraction terminate?

Reduce $\frac{p}{q}$ to lowest terms. It gives a **terminating** decimal *iff* the denominator’s only prime factors are 2 and/or 5 (i.e. $q = 2^m 5^n$). Otherwise it is **non-terminating recurring**. Any “foreign” prime

$(3, 7, 11, \dots)$ forces recurrence. Examples: $\frac{7}{20} = \frac{7}{2^2 \cdot 5} = 0.35$ terminates; $\frac{5}{12} = \frac{5}{2^2 \cdot 3} = 0.41\bar{6}$ recurs because of the factor 3.

Recurring decimal $\rightarrow$ fraction (the master rule)

For a bar over k digits, put k nines in the denominator; each *non-repeating* decimal digit after the point adds one 0 after the nines (equivalently a factor 10). In symbols

$$0.\overline{d_1 \dots d_k} = \frac{d_1 \dots d_k}{\underbrace{9 \dots 9}_k}, \quad 0.a_1 \dots a_t \overline{b_1 \dots b_k} = \frac{(a_1 \dots a_t b_1 \dots b_k) - (a_1 \dots a_t)}{\underbrace{9 \dots 9}_k \underbrace{0 \dots 0}_t}.$$

So $0.\overline{ab} = \frac{ab}{99}$ and $0.a\overline{b} = \frac{ab - a}{90}$ (one non-repeating digit $\Rightarrow$ one 0).

Short derivation of the mixed rule

Let $x = 0.58\bar{3}$. There is 1 non-repeating digit and 1 repeating digit. Multiply to shift the point past each block: $1000x = 583.\bar{3}$ and $100x = 58.\bar{3}$. Subtract: $(1000 - 100)x = 583 - 58$, so $900x = 525$ and $x = \frac{525}{900} = \frac{7}{12}$. The 900 = $9 \cdot 100$ is exactly “one 9 (for 1 repeat) followed by one 0 (for 1 non-repeat)” — the master rule in action.

$0.\bar{9} = 1$, not “just below 1”

$0.\bar{9} = \frac{9}{9} = 1$ exactly; it is another name for 1, not a number smaller than 1. Likewise $0.4\bar{9} = 0.5$. In the exam treat $0.\bar{9}$ as the integer 1 (a rational, and even a whole number).

Handy recurring conversions to memorise

$\frac{1}{3} = 0.\bar{3}$, $\frac{1}{6} = 0.1\bar{6}$, $\frac{1}{7} = 0.14285\bar{7}$, $\frac{1}{9} = 0.\bar{1}$, $\frac{1}{11} = 0.0\bar{9}$, $\frac{1}{12} = 0.08\bar{3}$, $\frac{1}{13} = 0.0\bar{7}692\bar{3}$. Reverse instantly: $0.4\bar{5} = \frac{45}{99} = \frac{5}{11}$.

1.5 Irrational numbers and the reals

Why $\sqrt{2}$ is irrational — the classic argument

Suppose, for contradiction, $\sqrt{2} = \frac{p}{q}$ in lowest terms ($\gcd(p, q) = 1$). Then $p^2 = 2q^2$, so p^2 is even, hence p is even, say $p = 2m$. Then $4m^2 = 2q^2 \Rightarrow q^2 = 2m^2$, so q is even too. But then $2 \mid p$ and $2 \mid q$ contradicts “lowest terms”. Hence no such $\frac{p}{q}$ exists: $\sqrt{2}$ is **irrational**. The same argument shows $\sqrt{3}, \sqrt{5}, \dots$ (roots of non-perfect squares) are irrational.

Operations on irrationals — what survives

- rational $\pm$ irrational = irrational (e.g. $3 + \sqrt{2}$, $\pi - \frac{22}{7}$).
- non-zero rational $\times$ irrational = irrational (e.g. $5\sqrt{2}$).
- irrational $\pm$ irrational *may be either*: $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$ (irrational) but $(3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$ (rational).

- irrational $\times$ irrational *may be either*: $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ (irrational) but $\sqrt{2} \cdot \sqrt{2} = 2$ and $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = 1$ (rational).

Spot the rational in disguise

Simplify before you classify: $\sqrt{45} \div \sqrt{5} = \sqrt{9} = 3$, $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$, $\sqrt{2.25} = 1.5$, $\frac{22}{7}$ is rational (a mere approximation to π , not π). The genuinely irrational classics are π , e , and surds of non-perfect powers.

1.6 Absolute value, integer ranges and standard form

Absolute value and counting integers

$|x|$ is the distance of x from 0: $|x| = x$ if $x \geq 0$, else $-x$. Thus $|x| \leq a \iff -a \leq x \leq a$ and $|x| \geq a \iff x \leq -a$ or $x \geq a$. **Counting integers** in $\ell \leq n \leq u$: the count is $\lfloor u \rfloor - \lceil \ell \rceil + 1$. For a *strict* bound, if an endpoint is itself an integer, drop it (move in by 1). Always test the two endpoints separately.

The boundary is where marks are won or lost

For $-\frac{20}{3} \leq n \leq \frac{31}{4}$: $-\frac{20}{3} = -6.\bar{6}$ forces $n \geq -6$ (since $-7 < -6.6$), and $\frac{31}{4} = 7.75$ forces $n \leq 7$. Count = $7 - (-6) + 1 = 14$. Draw the two boundary points and ask “does the nearest integer satisfy the sign?” — that one check removes the classic off-by-one error.

Standard (scientific) form

Any non-zero number = $k \times 10^n$ with $1 \leq k < 10$ and $n \in \mathbb{Z}$. Move the decimal point until one non-zero digit sits before it: moving *left* gives $n > 0$, moving *right* gives $n < 0$; n equals the number of places moved. Multiply/divide such forms by handling k and the powers of 10 separately (add exponents when multiplying, subtract when dividing), then re-normalise so that $1 \leq k < 10$.

$0.00001278 = 1.278 \times 10^{-5}$, so $k + n = 1.278 - 5 = -3.722$ (the 2025 board item, Q97). Similarly $45000 = 4.5 \times 10^4$ gives $k + n = 8.5$, and $(3.2 \times 10^{-3})(2.5 \times 10^6) = 8.0 \times 10^3$ gives $k + n = 11$.

Standard form needs $1 \leq k < 10$

12.5×10^3 and 0.7×10^{-2} are *not* in standard form. Re-normalise: $12.5 \times 10^3 = 1.25 \times 10^4$, $0.7 \times 10^{-2} = 7 \times 10^{-3}$. If you forget, the exponent n (and hence $k + n$) comes out wrong by 1.

1.7 Graded Solved Examples

Example 1.1 [Hard]

How many integers n satisfy $-\frac{20}{3} \leq n \leq \frac{31}{4}$?

Solution. $-\frac{20}{3} = -6.\bar{6}$ and $\frac{31}{4} = 7.75$. Since $-7 < -6.\bar{6}$, the smallest allowed integer is -6 ; since $8 > 7.75$, the largest is 7 . The integers $-6, -5, \dots, 7$ number $7 - (-6) + 1 = \boxed{14}$. **Trap:** $-6.\bar{6} \geq n$ does not admit -7 .

Example 1.2 [Hard]

How many integers n satisfy $-\frac{47}{5} < n < \frac{23}{4}$ (both strict)?

Solution. $-\frac{47}{5} = -9.4$ and $\frac{23}{4} = 5.75$. **Strict bounds and non-integer endpoints, so the integers run $-9, -8, \dots, 5$. Count $= 5 - (-9) + 1 = \boxed{15}$.** Because neither endpoint is an integer, strictness changes nothing here — but always check.

Example 1.3 [Exam-Level]

How many integers n satisfy $|2n - 3| \leq 7$?

Solution. $|2n - 3| \leq 7 \iff -7 \leq 2n - 3 \leq 7 \iff -4 \leq 2n \leq 10 \iff -2 \leq n \leq 5$. The integers $-2, -1, \dots, 5$ number $5 - (-2) + 1 = \boxed{8}$. **Reducing the absolute-value inequality to a double inequality is the whole trick.**

Example 1.4 [Exam-Level]

The product of all integers strictly between -7 and -3 is:

Solution. “Strictly between” excludes -7 and -3 ; the integers are $-6, -5, -4$. Three negative factors give a negative product: $(-6)(-5)(-4) = -120$. Answer $\boxed{-120}$. This is the 2025 Q93 board item; 360 forgets the sign and -140 is a miscount.

Example 1.5 [Hard]

Insert five rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$ by the step method.

Solution. Step $= \frac{b-a}{n+1} = \frac{\frac{3}{5} - \frac{2}{5}}{6} = \frac{1/5}{6} = \frac{1}{30}$. Add it repeatedly to $\frac{2}{5} = \frac{12}{30}$:

$$\frac{13}{30}, \frac{14}{30}, \frac{15}{30}, \frac{16}{30}, \frac{17}{30}.$$

So $\boxed{\frac{13}{30}, \frac{14}{30}, \frac{15}{30}, \frac{16}{30}, \frac{17}{30}}$. Check: the sixth step lands on $\frac{18}{30} = \frac{3}{5}$, the endpoint, confirming the spacing.

Example 1.6 [Hard]

Between $\frac{1}{3}$ and $\frac{1}{2}$, how many fractions have denominator 24?

Solution. Write both over 24: $\frac{1}{3} = \frac{8}{24}$, $\frac{1}{2} = \frac{12}{24}$. Strictly between 8 and 12 the numerators are 9, 10, 11, i.e. $\frac{9}{24} = \frac{3}{8}$, $\frac{10}{24} = \frac{5}{12}$, $\frac{11}{24}$. That is $\boxed{3}$ fractions. (If the question demanded lowest terms with denominator exactly 24, only $\frac{11}{24}$ qualifies.)

Example 1.7 [Exam-Level]

Convert the mixed recurring decimal $2.4\bar{5}$ to a fraction in lowest terms.

Solution. One non-repeating digit (4) and one repeating digit (5). Master rule: numerator $= (245) - (24) = 221$, denominator $= 90$ (one 9, one 0). So $2.4\bar{5} = \frac{221}{90}$, already in lowest terms ($\gcd(221, 90) = 1$). Answer $\boxed{\frac{221}{90}}$. Cross-check: $221 \div 90 = 2.4\bar{5}$.

Example 1.8 [Exam-Level]

Convert $0.58\bar{3}$ to a fraction, and hence classify $0.58\bar{3} + \sqrt{2}$.

Solution. By the mixed rule, numerator $= (583) - (58) = 525$, denominator $= 900$, so $0.58\bar{3} = \frac{525}{900} = \frac{7}{12}$ (rational). Then $\frac{7}{12} + \sqrt{2}$ is rational + irrational = $\boxed{\text{irrational}}$. A rational shift never removes the irrational part.

Example 1.9 [Exam-Level]

A number in lowest terms is $\frac{p}{q}$ with $q = 2^3 \cdot 3 \cdot 5^2$. Without dividing, decide whether its decimal terminates, and justify.

Solution. The denominator contains the prime 3, which is neither 2 nor 5. By the terminating rule the decimal is non-terminating recurring. (Had q been $2^3 \cdot 5^2 = 200$ it would terminate.) A single foreign prime is enough to force recurrence.

Example 1.10 [Exam-Level]

Classify each as rational or irrational: (i) $\sqrt{2} \cdot \sqrt{8}$, (ii) $\frac{\sqrt{45}}{\sqrt{5}}$, (iii) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$, (iv) $\pi - \frac{22}{7}$.

Solution. (i) $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$ — rational. (ii) $\frac{\sqrt{45}}{\sqrt{5}} = \sqrt{9} = 3$ — rational. (iii) $(\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$ — rational. (iv) π is irrational and $\frac{22}{7}$ is rational, so the difference is irrational. Moral: simplify first; only (iv) survives as irrational.

Example 1.11 [Exam-Level]

Write $(2.5 \times 10^{-4})(4 \times 10^7)$ in standard form and give $k + n$.

Solution. Multiply the k -parts and add exponents: $2.5 \times 4 = 10$ and $10^{-4} \times 10^7 = 10^3$, giving $10 \times 10^3 = 10^4 = 1 \times 10^4$ (re-normalised so $1 \leq k < 10$). Thus $k = 1$, $n = 4$ and $k + n = \boxed{5}$. Forgetting to re-normalise 10×10^3 would give the wrong $k = 10$.

Example 1.12 [Exam-Level — reasoning]

State with reasons whether each is True or False. (i) $\mathbb{W}$ is closed under subtraction. (ii) Every integer has a multiplicative inverse in $\mathbb{Z}$. (iii) The sum of two irrationals is always irrational. (iv) Between any two rationals there are only finitely many rationals.

Solution. (i) False: $3 - 5 = -2 \notin \mathbb{W}$. (ii) False: only ± 1 have inverses in $\mathbb{Z}$ (e.g. 2 has none). (iii) False: $(3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$ is rational. (iv) False: by density there are infinitely many. All four are “False” — a favourite examiner pattern, so read each clause on its own merits.

Previous-Year Questions (solved)

PYQ [2019, Q91] A rational number between 6 and 7 is

- (A) $\frac{12}{2}$ (B) $\frac{14}{2}$
(C) $\frac{13}{2}$ (D) None of the above

Answer & Solution. (C) $\frac{13}{2} = 6.5$, the average of 6 and 7, lies strictly between them.

Exam Insight. “Between” almost always means the **average**. Do not be rattled that the options are unreduced.

PYQ [2025, Q93] The product of integers between -7 and -3 is

- (A) -140 (B) -120
(C) 360 (D) None of the above

Answer & Solution. (B) -120 . The integers strictly between are $-6, -5, -4$; their product is $(-6)(-5)(-4) = -120$.

Exam Insight. An *odd* number of negative factors gives a negative product — decide the sign first, then the size.

PYQ [2025, Q97] If $0.00001278 = k \times 10^n$ is the standard form, then $k + n$ is

(A) 3.278

(B) -3.722

(C) 4.722

(D) None of the above

Answer & Solution. (B) $0.00001278 = 1.278 \times 10^{-5}$, so $k = 1.278$, $n = -5$ and $k + n = 1.278 - 5 = -3.722$.

Exam Insight. Count the shifts of the decimal point: right $\Rightarrow n < 0$. Here the first non-zero digit is the 5th place, so $n = -5$.

PYQ [2016, Q91] The difference between two numbers is 47.364. If the smaller number is 31.855, then the bigger number is

(A) 15.509

(B) 79.119

(C) 79.219

(D) 79.129

Answer & Solution. (C) bigger = smaller + difference = $31.855 + 47.364 = 79.219$.

Exam Insight. Line up the decimal points and add column-wise; the hundredths $5 + 4 = 9$ fix the last digit at once.

Expected & Probable Questions (board style)

Same standard as the real paper — not copied from any PYQ

E1. Which of the following is *not* closed under subtraction? (A) $\mathbb{Z}$ (B) $\mathbb{Q}$ (C) $\mathbb{N}$ (D) $\mathbb{R}$. **Ans (C)** — $2 - 5 = -3 \notin \mathbb{N}$.

E2. The number of rationals of the form $\frac{n}{7}$ lying strictly between 2 and 3 is (A) 5 (B) 6 (C) 7 (D) None.

Ans (B) — $\frac{15}{7} \dots \frac{20}{7}$, six values.

E3. $0.\overline{36}$ in lowest terms is (A) $\frac{11}{30}$ (B) $\frac{33}{90}$ (C) $\frac{11}{33}$ (D) None. **Ans (A)** — $\frac{36 - 3}{90} = \frac{33}{90} = \frac{11}{30}$.

E4. If $x = 1.\overline{27}$, then x equals (A) $\frac{14}{11}$ (B) $\frac{127}{99}$ (C) $\frac{126}{99}$ (D) None. **Ans (A)** — $1 + \frac{27}{99} = \frac{126}{99} = \frac{14}{11}$.

E5. The last digit of 7^{2025} is (A) 1 (B) 3 (C) 7 (D) 9. **Ans (C)** — the units cycle 7, 9, 3, 1 has period 4; $2025 \equiv 1 \pmod{4}$, giving 7.

Practice Set — Chapter 1

Q1. How many integers n satisfy $-\frac{47}{5} < n < \frac{23}{4}$?

(A) 16

(B) 17

(C) 15

(D) None

Q2. How many integers n satisfy $-\frac{13}{2} \leq n < \frac{17}{3}$?

(A) 11

(B) 12

(C) 13

(D) None

Q3. The number of integers with $|n| \leq \frac{9}{2}$ is

(A) 8

(B) 9

(C) 10

(D) None

- Q4.** If $2 < \frac{n}{3} < 5$ with $n \in \mathbb{Z}$, the number of values of n is
- (A) 7 (B) 8
(C) 9 (D) None
- Q5.** The number of *even* integers n with $-\frac{20}{3} \leq n \leq \frac{20}{3}$ is
- (A) 6 (B) 7
(C) 8 (D) None
- Q6.** How many integers n satisfy $-5 < n \leq \sqrt{50}$?
- (A) 11 (B) 12
(C) 13 (D) None
- Q7.** The number of fractions with denominator 24 lying strictly between $\frac{1}{3}$ and $\frac{1}{2}$ is
- (A) 2 (B) 3
(C) 4 (D) None
- Q8.** A rational number lying exactly midway between $\frac{5}{7}$ and $\frac{6}{7}$ is
- (A) $\frac{11}{7}$ (B) $\frac{11}{14}$
(C) $\frac{13}{14}$ (D) None
- Q9.** The number of rationals of the form $\frac{n}{6}$ lying strictly between 2 and 3 is
- (A) 4 (B) 5
(C) 6 (D) None
- Q10.** $0.58\overline{3}$ in lowest terms equals
- (A) $\frac{7}{12}$ (B) $\frac{525}{900}$
(C) $\frac{35}{60}$ (D) None
- Q11.** $2.4\overline{5}$ as a fraction in lowest terms is
- (A) $\frac{221}{90}$ (B) $\frac{245}{99}$
(C) $\frac{49}{20}$ (D) None
- Q12.** $0.1\overline{6}$ equals
- (A) $\frac{1}{6}$ (B) $\frac{15}{99}$
(C) $\frac{1}{60}$ (D) None

Q13. $1.\overline{245}$ as a fraction in lowest terms is

(A) $\frac{137}{110}$
(C) $\frac{123}{99}$

(B) $\frac{1245}{990}$
(D) None

Q14. If $0.\overline{12} = \frac{p}{q}$ in lowest terms, then $p + q$ equals

(A) 111
(C) 21

(B) 37
(D) None

Q15. If $x = 0.4\overline{7}$, then $90x$ equals

(A) 43
(C) 42

(B) 47
(D) None

Q16. Which of the following is irrational?

(A) $\sqrt{2.25}$
(C) $\pi - \frac{22}{7}$

(B) $\sqrt{2} \cdot \sqrt{8}$
(D) $\frac{\sqrt{45}}{\sqrt{5}}$

Q17. Which of the following is rational?

(A) $\sqrt{3} + \sqrt{2}$
(C) $\sqrt{2} + \pi$

(B) $(\sqrt{5} + 2)(\sqrt{5} - 2)$
(D) $\sqrt{7}$

Q18. $0.\overline{9}$ is

(A) just below 1
(C) irrational

(B) equal to 1
(D) None

Q19. Among $\mathbb{N}$, $\mathbb{W}$, $\mathbb{Z}$, the number of sets closed under subtraction is

(A) 0
(C) 2

(B) 1
(D) 3

Q20. The number of integers in $\mathbb{Z}$ possessing a multiplicative inverse in $\mathbb{Z}$ is

(A) 1
(C) infinitely many

(B) 2
(D) None

Q21. If $45000 = k \times 10^n$ in standard form, then $k + n$ equals

(A) 8.5
(C) 9.5

(B) 4.5
(D) None

Q22. If $0.00056 = k \times 10^n$ in standard form, then $k + n$ equals

(A) 1.6
(C) -2.4

(B) 5.6
(D) None

Q23. Writing $(3.2 \times 10^{-3})(2.5 \times 10^6)$ in standard form as $k \times 10^n$, $k + n$ equals

- (A) 8 (B) 11
(C) 13 (D) None

Q24. Writing $\frac{6.4 \times 10^5}{1.6 \times 10^2}$ in standard form as $k \times 10^n$, $k + n$ equals

- (A) 4 (B) 7
(C) 8 (D) None

Q25. The last digit of $2^{2024} + 3^{2024}$ is

- (A) 5 (B) 7
(C) 9 (D) None

Q26. The digital root (repeated digit-sum) of 9^{100} is

- (A) 1 (B) 3
(C) 9 (D) None

Q27. The largest of $\frac{5}{8}, \frac{7}{11}, \frac{9}{14}, \frac{2}{3}$ is

- (A) $\frac{5}{8}$ (B) $\frac{7}{11}$
(C) $\frac{9}{14}$ (D) $\frac{2}{3}$

Q28. The number of primes p with $\frac{1}{p}$ giving a *terminating* decimal is

- (A) 1 (B) 2
(C) infinitely many (D) None

Answer Key with reasons

1.(C) $-\frac{47}{5} = -9.4$, $\frac{23}{4} = 5.75 \Rightarrow n = -9, \dots, 5$; count $5 - (-9) + 1 = 15$.

2.(B) $-6.5 \leq n < 5.6 \Rightarrow n = -6, \dots, 5$; count $5 - (-6) + 1 = 12$.

3.(B) $|n| \leq 4.5 \Rightarrow -4 \leq n \leq 4$; count $4 - (-4) + 1 = 9$.

4.(A) $6 < n < 15 \Rightarrow n = 7, \dots, 14$; count $14 - 7 + 1 = 7$.

5.(B) $-6.\overline{6} \leq n \leq 6.\overline{6} \Rightarrow$ evens $-6, -4, -2, 0, 2, 4, 6 = 7$.

6.(B) $\sqrt{50} \approx 7.07 \Rightarrow -4 \leq n \leq 7$; count $7 - (-4) + 1 = 12$.

7.(B) $\frac{8}{24} < \cdot < \frac{12}{24} \Rightarrow$ numerators 9, 10, 11 = 3.

8.(B) average $= \frac{1}{2} \left(\frac{5}{7} + \frac{6}{7} \right) = \frac{11}{14}$.

9.(B) $\frac{12}{6} < \frac{n}{6} < \frac{18}{6} \Rightarrow n = 13, \dots, 17 = 5$ values.

10.(A) $\frac{583-58}{900} = \frac{525}{900} = \frac{7}{12}$ (lowest terms).

11.(A) $\frac{245-24}{90} = \frac{221}{90}$; $\gcd(221, 90) = 1$.

12.(A) $\frac{16-1}{90} = \frac{15}{90} = \frac{1}{6}$.

13.(A) $1 + \frac{245-2}{990} = 1 + \frac{243}{990} = 1 + \frac{27}{110} = \frac{137}{110}$.

14.(B) $0.\overline{12} = \frac{12}{99} = \frac{4}{33} \Rightarrow p + q = 4 + 33 = 37$.

15.(A) $x = \frac{47-4}{90} = \frac{43}{90} \Rightarrow 90x = 43$.

16.(C) $\pi - \frac{2}{7}$ is irrational; $\sqrt{2.25} = 1.5$, $\sqrt{2}\sqrt{8} = 4$, $\frac{\sqrt{45}}{\sqrt{5}} = 3$ are rational.

17.(B) $(\sqrt{5} + 2)(\sqrt{5} - 2) = 5 - 4 = 1$ rational; the others are irrational.

18.(B) $0.\overline{9} = \frac{9}{9} = 1$ exactly.

19.(B) Only $\mathbb{Z}$ is closed under subtraction $\Rightarrow 1$.

20.(B) Only $+1$ and -1 have inverses in $\mathbb{Z} \Rightarrow 2$.

- 21.(A) $45000 = 4.5 \times 10^4 \Rightarrow k + n = 4.5 + 4 = 8.5$.
 22.(A) $0.00056 = 5.6 \times 10^{-4} \Rightarrow k + n = 5.6 - 4 = 1.6$.
 23.(B) $8.0 \times 10^3 = 8 \times 10^3 \Rightarrow k + n = 8 + 3 = 11$.
 24.(B) $4.0 \times 10^3 = 4 \times 10^3 \Rightarrow k + n = 4 + 3 = 7$.
 25.(B) 2^{2024} ends 6, 3^{2024} ends 1 ($2024 \equiv 0 \pmod{4}$); $6 + 1 = 7$.
 26.(C) 9^{100} is a multiple of 9, so its digital root is 9.
 27.(D) cross-multiplying, $\frac{2}{3}$ beats all: $\frac{2}{3} > \frac{7}{11} > \frac{5}{8} > \frac{9}{14}$; answer (D) $\frac{2}{3}$.
 28.(B) $\frac{1}{p}$ terminates $\iff p \in \{2, 5\}$, exactly 2 primes.

One-Page Revision — Chapter 1

Formula & fact sheet.

- $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$; irrationals fill the gaps to make $\mathbb{R}$; simplify surds before classifying ($\sqrt{16} = 4$ is rational).
- Closure: $\mathbb{N}, \mathbb{W}$ under $+$, $\times$; $\mathbb{Z}$ also under $-$; $\mathbb{Q}, \mathbb{R}$ also under $\div$ (not by 0). $\mathbb{Q}'$ closed under none.
- Inverses: additive inverse from $\mathbb{Z}$ up; multiplicative inverse from $\mathbb{Q} \setminus \{0\}$ up; in $\mathbb{Z}$ only ± 1 ; 0 never.

- Between a, b : average $\frac{a+b}{2}$; n rationals via step $\frac{b-a}{n+1}$; **infinitely many** rationals (density).

Compare by cross-multiplying.

- $\frac{p}{q}$ (lowest terms) terminates $\iff q = 2^m 5^n$; any other prime $\Rightarrow$ recurring.

- $0.\overline{d_1 \dots d_k} = \frac{d_1 \dots d_k}{\underbrace{9 \dots 9}_k}$; mixed: $\frac{(\text{all}) - (\text{non-repeat})}{\underbrace{9 \dots 9}_k \underbrace{0 \dots 0}_t}$; $0.\overline{9} = 1$.

- Counting integers in $\ell \leq n \leq u$: $\lfloor u \rfloor - \lceil \ell \rceil + 1$; drop an integer endpoint under a strict bound. $|x| \leq a \iff -a \leq x \leq a$.

- Standard form $k \times 10^n$, $1 \leq k < 10$; $\times \Rightarrow$ add exponents, $\div \Rightarrow$ subtract, then re-normalise; $k + n$ is the usual board ask.

PYQ recap: rational-between (2019, avg), integer-product (2025, sign of negatives), standard-form $k + n$ (2025), decimal addition (2016).

CHAPTER 2

Playing with Numbers

LEARNING OBJECTIVES

After this chapter you will be able to: apply *every* divisibility rule (2–13, and composite tests such as $72 = 8 \times 9$, $88 = 8 \times 11$, $99 = 9 \times 11$) and prove why the digit-sum rules work; count the **number of factors** and compute their **sum** and **product** straight from the prime factorisation; use the **HCF–LCM** identity ($\text{HCF} \times \text{LCM} = \text{product of two numbers}$), Euclid’s division algorithm, and the meaning of *coprime*; find the **unit digit** of large powers by cyclicity; compute **remainders** of huge powers by reducing the base and cycling the exponent (with Fermat’s little theorem stated simply); count **trailing zeros** of products and of $n!$ using Legendre’s formula; apply **perfect-square / perfect-cube** last-digit and digital-root filters; and crack digit-puzzle (**cryptarithm**) questions — all at 2025 Olympiad rigour.

PYQ TREND ANALYSIS

PYQs from this chapter: ≈ 14 . **Board favourites:** divisibility with unknown digits ($\overline{134x58y}$ divisible by 72 — 2025 Q98), number of factors (36 in 2019, 100 in 2018), primes below 50 (2016), *non*-perfect-square by last digit (2015), digit-sum mod 9 identity (2016), unit digit of a sum of powers (2018-S2 Q92), trailing zeros of a product (2022 Q92), and remainder of a^n on division by a small number ($67^{99} \bmod 7$ — 2025 Q109). **2026 probability: very high** — expect at least one divisibility-with-digits or factor-count item, plus one unit-digit or remainder item; trailing zeros and cryptarithms are rising.

2.1 Divisibility rules — the complete toolkit

Single-prime and small rules

2: last digit even. **3:** digit sum divisible by 3. **4:** number formed by the last *two* digits divisible by 4. **5:** ends in 0 or 5. **6:** divisible by 2 *and* 3. **8:** number formed by the last *three* digits divisible by 8. **9:** digit sum divisible by 9. **10:** ends in 0. **11:** (sum of digits in odd places) – (sum of digits in even places) is 0 or a multiple of 11.

Rules for 7, 11, 13 — the “1001” family

Since $7 \times 11 \times 13 = 1001$, group the digits in **blocks of three from the right** and take the *alternating sum* of the blocks; the original number is divisible by 7, 11 or 13 exactly when that alternating sum is.

Standalone rule for 7: chop off the last digit d , subtract $2d$ from the rest, and repeat; $7 \mid N$ iff 7 divides the result. *For 13:* chop off d , add $4d$ to the rest.

Example 2.1 [Illustration — the block rule]

Test $N = 1\,092\,193$ for divisibility by 7, 11, 13. Blocks of three from the right: $1 \mid 092 \mid 193$. **Alternating sum** $= 193 - 092 + 1 = 102$. **Now $102 = 2 \cdot 3 \cdot 17$, which is *not* a multiple of 7, 11 or 13, so none divide N . (Contrast $1001 = 1 \mid 001$, alternating sum $001 - 1 = 0$, divisible by all three.)**

Composite divisors: split into coprime factors

To test a composite m , write $m = a \times b$ with a, b **coprime** and apply both rules:

$$72 = 8 \times 9, \quad 88 = 8 \times 11, \quad 99 = 9 \times 11, \quad 45 = 9 \times 5, \quad 36 = 4 \times 9, \quad 75 = 3 \times 25, \quad 12 = 3 \times 4.$$

Warning: the factors must be coprime. $12 \neq 2 \times 6$ for this purpose ($\gcd(2, 6) = 2$); use 3×4 .

The commonest divisibility trap

Splitting 72 as 9×8 is fine (coprime), but splitting 36 as 6×6 is *wrong* — 6 and 6 share the factor 6, so “ $6 \mid N$ twice” does *not* guarantee $36 \mid N$. Use 4×9 . Likewise never test 8 by “2 three times”; use the last-three-digits rule.

Why “number – digit sum” is always divisible by 9 — with proof

Write $N = \sum d_k 10^k$. Since $10 \equiv 1 \pmod{9}$, we get $10^k \equiv 1^k = 1 \pmod{9}$, so

$$N = \sum d_k 10^k \equiv \sum d_k = (\text{digit sum}) \pmod{9}.$$

Hence $N - (\text{digit sum}) \equiv 0 \pmod{9}$ *always* — the 2016 board answer. The same congruence $10 \equiv 1 \pmod{3}$ proves the $\div 3$ rule, and $10 \equiv -1 \pmod{11}$ (so $10^k \equiv (-1)^k$) proves the alternating-sum rule for 11.

2.2 HCF, LCM and Euclid’s algorithm

Definitions and the master identity

For positive integers a, b with prime factorisations, the **HCF** (GCD) takes the *lowest* power of each common prime and the **LCM** takes the *highest* power of every prime. The master identity (for *two* numbers only) is

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

Two numbers are **coprime** (relatively prime) when $\text{HCF} = 1$; then $\text{LCM} = a \times b$.

The identity fails for three numbers

$\text{HCF} \times \text{LCM} = \text{product}$ holds *only* for two numbers. For a, b, c it is generally false: e.g. $a = b = c = 2$ gives $\text{HCF} = 2$, $\text{LCM} = 2$, product of the three = $8 \neq 2 \times 2 = 4$.

Euclid’s division algorithm

Every pair $a > b > 0$ satisfies $a = bq + r$ with $0 \leq r < b$. Repeatedly replacing (a, b) by (b, r) until the remainder is 0 gives $\text{HCF}(a, b)$ as the last non-zero remainder.

Example. $\text{HCF}(1651, 2032)$: $2032 = 1 \cdot 1651 + 381$; $1651 = 4 \cdot 381 + 127$; $381 = 3 \cdot 127 + 0$. Last non-zero remainder = **127**.

Word-problem signals

“Greatest number that divides ... leaving remainders r_1, r_2 ” $\Rightarrow$ take HCF of $(N_1 - r_1), (N_2 - r_2)$.
“Least number, or bells tolling together again” $\Rightarrow$ take LCM. “Largest tile / greatest length” $\Rightarrow$ HCF; “smallest square / shortest common time” $\Rightarrow$ LCM.

2.3 Factors: how many, and their sum & product

Everything from the prime factorisation

If $N = p^a q^b r^c \dots$ then, writing d for the number of factors,

$$\text{number of factors } d = (a+1)(b+1)(c+1)\dots, \quad \text{sum of factors} = \frac{p^{a+1}-1}{p-1} \cdot \frac{q^{b+1}-1}{q-1} \dots,$$

$$\text{product of factors} = N^{d/2}.$$

Odd, even and square factors

Write $N = 2^a \cdot M$ with M odd. Then: **odd factors** = number of factors of M (ignore the 2); **even factors** = $d - (\text{odd factors})$. A factor is a **perfect square** iff every prime exponent chosen is even, so the count of square factors multiplies $(\lfloor a/2 \rfloor + 1)$ over all primes.

Example 2.2 [Worked count]

$36 = 2^2 \cdot 3^2 \Rightarrow d = (2+1)(2+1) = 9$; **sum** = $\frac{2^3-1}{1} \cdot \frac{3^3-1}{2} = 7 \cdot 13 = 91$; **product** = $36^{9/2} = 36^{4.5} = 6^9$.
 $100 = 2^2 \cdot 5^2 \Rightarrow d = 9$ **as well — the “twin” of 36.**

Primes to memorise

There are exactly 15 **primes below 50**: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47; and 25 **primes below 100** (the extra 10 are 53, 59, 61, 67, 71, 73, 79, 83, 89, 97). Note 1 is *not* prime, and 2 is the only even prime.

2.4 Unit digit by cyclicity

Cycle lengths of the last digit

As the power grows, the unit digit repeats:

2: (2, 4, 8, 6), 3: (3, 9, 7, 1), 7: (7, 9, 3, 1), 8: (8, 4, 2, 6) have period 4;

4: (4, 6), 9: (9, 1) have period 2; 0, 1, 5, 6 are constant.

Method: keep only the unit digit of the base; reduce the exponent modulo the period (use the period itself, not 0, when it divides exactly). For period 4: exponent $\equiv 1, 2, 3, 0$ picks the 1st, 2nd, 3rd, 4th entry.

The “mod 4” shortcut

For any base ending in 2, 3, 7, 8, look at the exponent mod 4. Since $2^{2024} \equiv 0$, 2^{2024} ends in 6 and 3^{2024} ends in 1. A tower like 7^{7^7} needs the *exponent* $7^7 \bmod 4$: as $7 \equiv -1$, $7^7 \equiv -1 \equiv 3$, so 7^{7^7} ends like $7^3 = 343$, i.e. in 3.

2.5 Remainders of large powers

Two-step reduction

To find $a^n \bmod m$: (1) replace a by $a \bmod m$; (2) find the smallest k with $a^k \equiv 1 \pmod{m}$ (the *order*) and reduce the exponent modulo k . If a factor is $\equiv -1$, an *odd* power keeps the -1 and an even power gives $+1$.

Fermat's little theorem, stated simply

If p is prime and $p \nmid a$, then $a^{p-1} \equiv 1 \pmod{p}$. So powers of a repeat with period dividing $p-1$. Example: $67 \equiv 4 \pmod{7}$ and $4^3 = 64 \equiv 1$, so $67^{99} = (4^3)^{33} \equiv 1 \pmod{7}$: the remainder is 1 (2025 Q109).

A powerful cancellation

If $m \mid (a+b)$ and n is odd, then $m \mid (a^n + b^n)$ because $a \equiv -b$ gives $a^n \equiv (-b)^n = -b^n$. Thus $15^{23} + 23^{23}$ is divisible by $15 + 23 = 38$ (and hence by 19): the remainder mod 19 is 0.

2.6 Trailing zeros

Zeros come from $10 = 2 \times 5$

The number of trailing zeros of an integer = $\min(\#2\text{'s}, \#5\text{'s})$ in its prime factorisation. In a product, count the total power of 2 and of 5 and take the minimum.

Legendre's formula for $n!$

The exponent of a prime p in $n!$ is $\left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \dots$. Because 5's are scarcer than 2's, the number of **trailing zeros** of $n!$ is

$$Z(n) = \left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{25} \right\rfloor + \left\lfloor \frac{n}{125} \right\rfloor + \dots$$

E.g. $Z(100) = 20 + 4 = 24$, $Z(60) = 12 + 2 = 14$.

Zeros of a binomial coefficient

Count 5's and 2's in numerator minus denominator separately. $\binom{60}{30}$ has *equal* scarce powers top and bottom, so it ends in 0 zeros; but $\binom{200}{100}$ has exactly **one** trailing zero (the 5-count exceeds the 2-count by one).

2.7 Perfect squares, cubes and digital roots

Last-digit and digital-root filters

A **perfect square** never ends in 2, 3, 7, 8; its possible last digits are 0, 1, 4, 5, 6, 9, and its *digital root* (repeated digit sum) is one of 1, 4, 7, 9. A **perfect cube** may end in any digit, but the last digit determines the cube-root's last digit uniquely ($\dots 0 \rightarrow 0$, $1 \rightarrow 1$, $8 \rightarrow 2$, $7 \rightarrow 3$, $4 \rightarrow 4$, $5 \rightarrow 5$, $6 \rightarrow 6$, $3 \rightarrow 7$, $2 \rightarrow 8$, $9 \rightarrow 9$).

Digital root = $1 + (n-1) \bmod 9$

The digital root of N equals $N \bmod 9$ (with 9 replacing 0). Squares have digital root 1, 4, 7, 9; a number with digital root 2, 3, 5, 6, 8 cannot be a perfect square. Useful for powers too: $7^{100} \equiv 7 \pmod{9}$, so its digital root is 7 — immediately *not* a perfect square.

2.8 Cryptarithms (digit puzzles)

Rules of the game

Each distinct letter is a distinct digit 0–9; no leading digit is 0. Attack the units column first, carry upward, and use divisibility/parity to prune. Palindromic sums help: $\overline{AB} + \overline{BA} = 10A + B + 10B + A = 11(A + B)$, always a multiple of 11; and the repeat number $\overline{abcabc} = \overline{abc} \times 1001 = \overline{abc} \times 7 \times 11 \times 13$.

2.9 Graded Solved Examples

Example 2.3 [Hard]

Find $(2x + y)$ if the 7-digit number $\overline{134x58y}$ is divisible by 72.

Solution. $72 = 8 \times 9$. Div by 8: last three digits $\overline{58y}$ divisible by 8; $580 = 8 \cdot 72 + 4$, so $4 + y \equiv 0 \pmod{8} \Rightarrow y = 4$. Div by 9: digit sum $1+3+4+x+5+8+4 = 25 + x \equiv 0 \pmod{9} \Rightarrow x = 2$. Thus $2x + y = 2(2) + 4 = \boxed{8}$.

Example 2.4 [Hard]

The largest four-digit number exactly divisible by 88 is:

Solution. Largest four-digit number = 9999; $9999 \div 88 = 113.6\dots$, and $88 \times 113 = 9944$. Remainder $9999 - 9944 = 55$, so the answer is $9999 - 55 = \boxed{9944}$.

Example 2.5 [Exam-Level]

Find the unit digit of $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$.

Solution. Use unit digits: $1^1=1$; $2^2=4$; 3^3 (cycle 3, 9, 7, 1) ends 7; 4^4 (cycle 4, 6, even power) ends 6; 5^5 ends 5; 6^6 ends 6. Sum = $1+4+7+6+5+6 = 29 \Rightarrow$ unit digit $\boxed{9}$.

Example 2.6 [Exam-Level]

How many trailing zeros are in $P = 5 \times 10 \times 25 \times 40 \times 50 \times 55 \times 65 \times 125 \times 80$?

Solution. Zeros = $\min(\#2\text{'s}, \#5\text{'s})$. Twos: $10=2 \cdot 5$, $40=2^3 \cdot 5$, $50=2 \cdot 5^2$, $80=2^4 \cdot 5$ give $1+3+1+4 = 9$. Fives: $5, 10, 25=5^2, 40, 50=5^2, 55, 65, 125=5^3, 80$ give $1+1+2+1+2+1+1+3+1 = 13$. Zeros = $\min(9, 13) = \boxed{9}$.

Example 2.7 [Hard]

Find the remainder when 2^{100} is divided by 7.

Solution. $2^3 = 8 \equiv 1 \pmod{7}$, so the powers of 2 cycle with period 3. Since $100 = 3 \cdot 33 + 1$, $2^{100} = (2^3)^{33} \cdot 2^1 \equiv 1 \cdot 2 = \boxed{2}$.

Example 2.8 [Hard]

The largest four-digit number that leaves remainder 3 when divided by each of 7, 9, 11 is:

Solution. The number is 3 more than a common multiple, so it has the form $693k + 3$ (since $\text{lcm}(7, 9, 11) = 693$). The largest such ≤ 9999 : $693 \times 14 = 9702$, giving $9702 + 3 = \boxed{9705}$. Check: $9705 - 3 = 9702 = 693 \times 14$, divisible by 7, 9, 11.

Example 2.9 [Exam-Level]

How many factors of 10800 are odd, and how many are even?

Solution. $10800 = 2^4 \cdot 3^3 \cdot 5^2$. Total factors = $(4+1)(3+1)(2+1) = 60$. Odd factors ignore the 2: $(3+1)(2+1) = 12$. Even = $60 - 12 = \boxed{48}$.

Example 2.10 [Hard]

The trailing zeros of $60!$ are how many? And of $100!$?

Solution. By Legendre, $Z(60) = \lfloor 60/5 \rfloor + \lfloor 60/25 \rfloor = 12 + 2 = \boxed{14}$; and $Z(100) = \lfloor 100/5 \rfloor + \lfloor 100/25 \rfloor = 20 + 4 = \boxed{24}$. (No $\lfloor n/125 \rfloor$ term since $125 > 100$.)

Example 2.11 [Exam-Level, HCF–LCM word problem]

The HCF of two numbers is 12 and their LCM is 336. If one number is 48, find the other.

Solution. By the identity $\text{HCF} \times \text{LCM} = \text{product}$, the other $= \frac{12 \times 336}{48} = \frac{4032}{48} = \boxed{84}$. Check: $\text{HCF}(48, 84) = 12$, $\text{LCM}(48, 84) = 336$.

Example 2.12 [Hard, HCF–LCM word problem]

The sum of two numbers is 216 and their HCF is 27. How many such pairs of numbers exist?

Solution. Write the numbers as $27a$ and $27b$ with $\gcd(a, b) = 1$. Then $27(a + b) = 216 \Rightarrow a + b = 8$. Coprime pairs with $a < b$ and $a + b = 8$: $(1, 7)$ and $(3, 5)$ — while $(2, 6)$, $(4, 4)$ fail (common factor). So there are $\boxed{2}$ pairs, namely $(27, 189)$ and $(81, 135)$.

Example 2.13 [Exam-Level, remainder cancellation]

Find the remainder when $2^{70} + 3^{70}$ is divided by 13.

Solution. Work mod 13: $2^6 = 64 \equiv 12 \equiv -1$, so $2^{70} = 2^{6 \cdot 11 + 4} \equiv (-1)^{11} 2^4 = -16 \equiv -3$. Also $3^3 = 27 \equiv 1$, so $3^{70} = 3^{3 \cdot 23 + 1} \equiv 3$. Sum $\equiv -3 + 3 = 0$: remainder $\boxed{0}$.

Example 2.14 [Hard, cryptarithm]

In the addition $\overline{AB} + \overline{BA} = 132$ with $A > B$, find $A + B$ and one valid pair (A, B) .

Solution. $\overline{AB} + \overline{BA} = 11(A + B) = 132 \Rightarrow A + B = \boxed{12}$. With $A > B$ and single digits, $(A, B) = (9, 3), (8, 4), (7, 5)$ all work (e.g. $93 + 39 = 132$). The puzzle fixes only the sum $A + B = 12$.

Previous-Year Questions (solved)

PYQ [2019, Q92] The total number of factors of the number 36 is

- (A) 8 (B) 9
(C) 10 (D) None of the above

Answer & Solution. (B) $36 = 2^2 \cdot 3^2 \Rightarrow (2+1)(2+1) = 9$.

Exam Insight. Always add 1 to *each* exponent, then multiply.

PYQ [2018-S1, Q113] The total number of factors of 100 is

- (A) 9 (B) 10
(C) 8 (D) None

Answer & Solution. (A) $100 = 2^2 \cdot 5^2 \Rightarrow (2+1)(2+1) = 9$.

Exam Insight. 36 and 100 are “twin” factor questions — both give 9.

PYQ [2016, Q92] If the digit-sum of a number between 100 and 1000 is subtracted from the number, the difference is always divisible by

- (A) 9 (B) 12
(C) 6 (D) None

Answer & Solution. (A) Every number $\equiv$ its digit sum (mod 9) (since $10 \equiv 1$), so the difference $\equiv 0$ (mod 9).

Exam Insight. The phrase “always divisible” signals a mod9 identity.

PYQ [2016, Q93] The number of prime numbers less than 50 is

- (A) 13 (B) 14
(C) 15 (D) 16

Answer & Solution. (C) 15 primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

Exam Insight. Memorise “15 below 50, 25 below 100.”

PYQ [2015, Q98] Which of the following is *not* a perfect square?

- (A) 34692 (B) 15129
(C) 4096 (D) 2025

Answer & Solution. (A) A perfect square never ends in 2, 3, 7, 8. 34692 ends in 2 $\Rightarrow$ not a square. (15129 = 123², 4096 = 64², 2025 = 45².)

Exam Insight. Apply the *last-digit* filter first — it settles most such questions instantly.

PYQ [2025, Q98] The 7-digit number $\overline{134x58y}$ is divisible by 72. Then $2x + y$ equals

- (A) 8 (B) 9
(C) 6 (D) None

Answer & Solution. (A) $72 = 8 \times 9$. Div by 8: $580 + y \equiv 0 \pmod{8} \Rightarrow y = 4$. Div by 9: $25 + x \equiv 0 \pmod{9} \Rightarrow x = 2$. So $2x + y = 8$.

Exam Insight. For a 72-question always split into the coprime pair 8 and 9.

PYQ [2025, Q109] The remainder when 67^{99} is divided by 7 is

- (A) 1 (B) 2
(C) 4 (D) None

Answer & Solution. (A) $67 \equiv 4 \pmod{7}$ and $4^3 = 64 \equiv 1$, so $4^{99} = (4^3)^{33} \equiv 1$.

Exam Insight. Reduce the base first, then use Fermat: $4^6 \equiv 1$, and here even $4^3 \equiv 1$.

PYQ [2022, Q92] The number of zeros at the end of $5 \times 10 \times 25 \times 40 \times 50 \times 55 \times 65 \times 125 \times 80$ is

- (A) 8 (B) 9
(C) 12 (D) None

Answer & Solution. (B) Twos = 9, fives = 13; trailing zeros = $\min(9, 13) = 9$.

Exam Insight. Zeros come from 2 $\times$ 5 pairs — the *scarcer* prime (here 2) wins.

PYQ [2018-S2, Q92] The unit digit of $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$ is

- (A) 9 (B) 0
(C) 1 (D) None

Answer & Solution. (A) Unit digits 1, 4, 7, 6, 5, 6 sum to 29; unit digit = 9.

Exam Insight. Reduce each base to its unit digit and its exponent mod (period), then add.

Expected & Probable Questions

Board-standard, freshly set — with answers

E1. Remainder when 2^{100} is divided by 7: (A) 1 (B) 2 (C) 4 (D) None. (B) — $2^3 \equiv 1$, $100 \equiv 1 \pmod{3}$, so $2^{100} \equiv 2$.

E2. If $\overline{7x36y5}$ is divisible by 75, the number of valid (x, y) pairs is (A) 2 (B) 4 (C) 6 (D) None. (C) — div by 25 needs $\overline{y5} \in \{25, 75\}$ so $y \in \{2, 7\}$; div by 3 then allows $x \in \{1, 4, 7\}$ for each, giving 6 pairs.

E3. Unit digit of 7^{85} : (A) 7 (B) 3 (C) 9 (D) 1. (A) — cycle (7, 9, 3, 1), $85 \equiv 1 \pmod{4}$.

E4. Number of trailing zeros in $100!$: (A) 20 (B) 24 (C) 25 (D) None. (B) $= \lfloor 100/5 \rfloor + \lfloor 100/25 \rfloor = 24$.

E5. A number leaves remainder 6 on dividing 1657 and remainder 5 on dividing 2037; the greatest such divisor is (A) 127 (B) 133 (C) 381 (D) None. (A) $= \gcd(1651, 2032) = 127$.

Practice Set — Chapter 2

Q1. The 4-digit number $\overline{4x2y}$ is divisible by 45. The number of valid (x, y) pairs is

- | | |
|-------|----------|
| (A) 1 | (B) 2 |
| (C) 3 | (D) None |

Q2. If the 6-digit number $\overline{8a459b}$ is divisible by 88, then $a + b$ equals

- | | |
|-------|----------|
| (A) 5 | (B) 6 |
| (C) 8 | (D) None |

Q3. The total number of factors of 7560 is

- | | |
|--------|----------|
| (A) 48 | (B) 64 |
| (C) 60 | (D) None |

Q4. The number of *odd* factors of 10800 is

- | | |
|--------|----------|
| (A) 12 | (B) 15 |
| (C) 24 | (D) None |

Q5. The unit digit of $2^{2024} + 3^{2024}$ is

- | | |
|-------|----------|
| (A) 5 | (B) 7 |
| (C) 1 | (D) None |

Q6. The unit digit of 7^{7^7} is

- | | |
|-------|----------|
| (A) 7 | (B) 3 |
| (C) 9 | (D) None |

Q7. The remainder when 3^{100} is divided by 5 is

- | | |
|-------|----------|
| (A) 1 | (B) 3 |
| (C) 4 | (D) None |

Q8. The remainder when $15^{23} + 23^{23}$ is divided by 19 is

- | | |
|--------|----------|
| (A) 0 | (B) 1 |
| (C) 18 | (D) None |

Q9. The remainder when $2^{70} + 3^{70}$ is divided by 13 is

- | | |
|--------|----------|
| (A) 0 | (B) 1 |
| (C) 12 | (D) None |

Q10. The number of trailing zeros in $60!$ is

- | | |
|--------|----------|
| (A) 12 | (B) 14 |
| (C) 15 | (D) None |

Q11. The number of trailing zeros in $\binom{200}{100}$ is

- | | |
|-------|----------|
| (A) 0 | (B) 1 |
| (C) 2 | (D) None |

Q12. The highest power of 3 contained in $100!$ is

- | | |
|--------|----------|
| (A) 33 | (B) 48 |
| (C) 50 | (D) None |

Q13. The number of zeros at the end of $8 \times 15 \times 24 \times 32 \times 45 \times 60$ is

- | | |
|-------|----------|
| (A) 3 | (B) 4 |
| (C) 5 | (D) None |

Q14. The HCF of two numbers is 12 and their LCM is 336; if one number is 48, the other is

- | | |
|--------|----------|
| (A) 72 | (B) 84 |
| (C) 96 | (D) None |

Q15. The sum of two numbers is 216 and their HCF is 27. The number of such pairs is

- | | |
|-------|----------|
| (A) 1 | (B) 2 |
| (C) 3 | (D) None |

Q16. The largest 4-digit number leaving remainder 3 when divided by each of 7, 9, 11 is

- | | |
|----------|----------|
| (A) 9705 | (B) 9702 |
| (C) 9999 | (D) None |

Q17. The greatest number dividing 1657 and 2037 leaving remainders 6 and 5 respectively is

- | | |
|---------|----------|
| (A) 127 | (B) 133 |
| (C) 254 | (D) None |

Q18. Five bells toll at intervals of 6, 7, 8, 9, 12 seconds. In 30 minutes, after the first simultaneous toll, they toll together again

- | | |
|-------------|-------------|
| (A) 2 times | (B) 3 times |
| (C) 4 times | (D) None |

Q19. Which of the following is *not* a perfect square?

- | | |
|-----------|-----------|
| (A) 15876 | (B) 11025 |
| (C) 46000 | (D) None |

Q20. Which of the following is *not* a perfect cube?

- | | |
|-----------|-----------|
| (A) 13824 | (B) 32768 |
| (C) 45000 | (D) None |

Q21. The digital root of 7^{100} is

- | | |
|-------|----------|
| (A) 1 | (B) 7 |
| (C) 9 | (D) None |

Q22. In the cryptarithm $\overline{AB} + \overline{BA} = 132$ (with $A > B$), the value of $A + B$ is

- (A) 11 (B) 12
(C) 13 (D) None

Q23. Every 6-digit number of the form $\overline{abcabc}$ is always divisible by

- (A) 1001 (B) 101
(C) 111 (D) None

Q24. The number of trailing zeros in $1^1 \times 2^2 \times 3^3 \times \cdots \times 10^{10}$ is

- (A) 10 (B) 15
(C) 20 (D) None

Answer Key with reasons

- 1.(B) $45 = 9 \times 5$; div by 5 $\Rightarrow y \in \{0, 5\}$, div by 9 $\Rightarrow$ digit sum $6+x+y \equiv 0 \pmod 9$. Only $(x, y) = (3, 0)$ and $(7, 5)$ work $\Rightarrow 2$ pairs.
- 2.(A) $88 = 8 \times 11$; last three $\overline{59b}$ div by 8 $\Rightarrow b = 2$; alternating sum $(8-a+4-5+9-2) = 14-a \equiv 0 \pmod{11} \Rightarrow a = 3$. So $a+b=5$.
- 3.(B) $7560 = 2^3 \cdot 3^3 \cdot 5 \cdot 7 \Rightarrow (3+1)(3+1)(1+1)(1+1) = 64$.
- 4.(A) $10800 = 2^4 \cdot 3^3 \cdot 5^2$; odd factors ignore the 2: $(3+1)(2+1) = 12$.
- 5.(B) $2024 \equiv 0 \pmod 4 \Rightarrow 2^{2024}$ ends 6, 3^{2024} ends 1; $6+1=7$.
- 6.(B) Exponent $7^7 \pmod 4$: $7 \equiv -1 \Rightarrow 7^7 \equiv 3$; $7^3 = 343$ ends 3.
- 7.(A) $3^4 \equiv 1 \pmod 5$; $100 \equiv 0 \pmod 4 \Rightarrow 3^{100} \equiv 1$.
- 8.(A) 23 is odd and $19 \mid (15+23) = 38$, so $19 \mid (15^{23} + 23^{23})$; remainder 0.
- 9.(A) $\pmod{13}$: $2^{70} \equiv -3$ and $3^{70} \equiv 3$ (since $3^3 \equiv 1$); sum $\equiv 0$.
- 10.(B) $\lfloor 60/5 \rfloor + \lfloor 60/25 \rfloor = 12 + 2 = 14$.
- 11.(B) Power of 5 in $200!$ is $40+8+1 = 49$, in each $100!$ is 24; net 5's = $49 - 48 = 1$, and 2's net ≥ 1 , so exactly 1 zero.
- 12.(B) $\lfloor 100/3 \rfloor + \lfloor 100/9 \rfloor + \lfloor 100/27 \rfloor + \lfloor 100/81 \rfloor = 33 + 11 + 3 + 1 = 48$.
- 13.(A) Product = $2^{11} \cdot 3^4 \cdot 5^3 \cdot \dots$; #2 = 11, #5 = 3 $\Rightarrow \min = 3$.
- 14.(B) Other = $\frac{12 \times 336}{48} = 84$.
- 15.(B) $27(a+b) = 216 \Rightarrow a+b = 8$; coprime $(1, 7), (3, 5) \Rightarrow 2$ pairs.
- 16.(A) $\text{lcm}(7, 9, 11) = 693$; $693 \times 14 + 3 = 9705 \leq 9999$.
- 17.(A) $\text{gcd}(1657-6, 2037-5) = \text{gcd}(1651, 2032) = 127$.
- 18.(B) $\text{lcm}(6, 7, 8, 9, 12) = 504$ s; $1800 \div 504 = 3$ full intervals $\Rightarrow 3$ times.
- 19.(C) 46000 ends in 000 but $\sqrt{46000} \approx 214.5$ is non-integer; $15876 = 126^2$, $11025 = 105^2$ are squares.
- 20.(C) 45000 is not a cube; $13824 = 24^3$, $32768 = 32^3$ are cubes.
- 21.(B) Digital root = $7^{100} \pmod 9$; $7^3 \equiv 1 \Rightarrow 7^{100} \equiv 7$.
- 22.(B) $\overline{AB} + \overline{BA} = 11(A+B) = 132 \Rightarrow A+B = 12$.
- 23.(A) $\overline{abcabc} = \overline{abc} \times 1001$, so always divisible by 1001 ($= 7 \cdot 11 \cdot 13$).
- 24.(B) Power of 5 = $5^5 \cdot 10^{10} \Rightarrow 5+10 = 15$ fives; 2's far exceed 15; zeros = 15.

One-Page Revision — Chapter 2

- **Div rules:** 4 $\rightarrow$ last 2, 8 $\rightarrow$ last 3, 9 $\rightarrow$ digit sum, 11 $\rightarrow$ alternating sum, 7/11/13 $\rightarrow$ alternating sum of 3-digit blocks ($1001 = 7 \cdot 11 \cdot 13$); composite $\rightarrow$ split into *coprime* factors ($72=8 \times 9$, $88=8 \times 11$).
- **N-digitsum** $\equiv 0 \pmod 9$ always, because $10 \equiv 1 \pmod 9$.
- **Factors:** $N = p^a q^b \cdots \Rightarrow d = (a+1)(b+1) \dots$; sum = $\prod \frac{p^{a+1}-1}{p-1}$; product = $N^{d/2}$; odd factors ignore the 2.
- **HCF $\times$ LCM = product** (two numbers only); coprime $\Leftrightarrow \text{HCF} = 1$; Euclid: last non-zero remainder.
- **Primes:** 15 below 50, 25 below 100; 1 not prime, 2 the only even prime.
- **Unit digit:** period-4 for 2, 3, 7, 8 (reduce exponent mod 4); period-2 for 4, 9; 0, 1, 5, 6 fixed.
- **Remainders:** reduce base mod m , then cycle the exponent (Fermat: $a^{p-1} \equiv 1$); $m \mid (a+b)$ with

odd $n \Rightarrow m \mid (a^n + b^n)$.

- **Trailing zeros:** $\min(\#2, \#5)$ in a product; $Z(n!) = \lfloor n/5 \rfloor + \lfloor n/25 \rfloor + \dots$

- **Squares** never end in 2, 3, 7, 8; digital root $\in \{1, 4, 7, 9\}$. Cubes end in all digits.

PYQ recap: factors of 36/100 (2019/18), primes < 50 (2016), digit-sum mod 9 (2016), non-square by last digit (2015), div-by-72 $\Rightarrow 2x+y=8$ (2025), $67^{99} \bmod 7 = 1$ (2025), zeros of a product = 9 (2022), unit digit of a power-sum = 9 (2018-S2).